THE RELAXATION TIME OF AIR IN THUNDERSTORMS

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ABSTRACT

The concept of how the conducting atmosphere in various states of ionization can lead to various relaxation times for changes in the observed electric field is investigated. It is concluded that the observed relaxation times can be short, and that the short relaxation times will exist even though Ohm's law is not strictly obeyed in relations between the current density and the electric field.

1. INTRODUCTION

If the charge carriers in a conduction current are produced by strong electric fields, some observers argue that the concept of conductivity is no longer strictly operative and that short relaxation times do not necessarily imply a large conductivity. Many of the concepts of electrical conduction, however, are still operative, and they can lead to a relaxation time for the observed changes in the electric field as described by Gunn [6]. Conductivity is defined through Ohm's law as $\mathbf{j}=\lambda\mathbf{E}$ where $\mathbf{j}$ is the current density and $\mathbf{E}$ is the electric field. If λ is in any way a function of $\mathbf{E}$, there will no longer be a linear relation between $\mathbf{j}$ and $\mathbf{E}$, and it is the purpose of this paper to investigate how the non-linear relation may be reflected in the observed relaxation time.

2. CONDUCTIVITY VARYING WITH TIME

From the microscopic point of view the current density, $\mathbf{j}$, is written as

$$\mathbf{j}=n_+e_+\mathbf{v}_+ + n_-e_-\mathbf{v}_-,$$

where $n_\pm$ is the number of ions per unit volume, $e_\pm$ is their charge, and $\mathbf{v}_\pm$ is the velocity. Usually $\mathbf{v}$ is proportional to the electric field $\mathbf{E}$, and the expression leads to a well-defined microscopic concept of conductivity; namely,

$$\lambda=(n_+e_+\omega_+ + n_-e_-\omega_-)$$

where $\omega_\pm$ is the dynamic mobility of the ion due to the force field produced by $\mathbf{E}$.

The values of $n_\pm$ are determined by kinetic equilibrium between production rates per unit volume, q, the recombination of the ions, and the combination of the ions with other immobile particles, which may change the dynamic mobility of the ions by many orders of magnitude. If any of these quantities change, the value of $n_\pm$ will change, but it will take some time for this to be realized. As long as there are ions present in the field, the ions will neutralize the charges which produce the electric fields.

Rather than consider how the electric conductivity may vary with the electric field, it will be better to consider how the conductivity varies with time even though the time variation may be caused by field changes.

If there is a charge Q which produces an electric field $\mathbf{E}$ in a conducting medium, then

$$\frac{dQ}{dt}=-\oint\!\!\oint \lambda\mathbf{E}\cdot d\mathbf{A},$$

where $\mathbf{A}$ is the area surrounding the charge. Since the present discussion involves a study of time variation, it will be assumed that there is spatial symmetry, in which case the equation reduces to

$$\frac{dQ}{dt}=-4\pi\lambda Q.$$

The solution then is

$$Q=Q_0\exp\left(-\int_0^t 4\pi\lambda dt\right)$$

where Q_0 is the value of the charge at zero time. If λ varies with time, it will be possible to investigate the deviation from simple exponential decay.

For the purposes of this analysis it will suffice to consider the ion production and recombination equation in the form

$$\frac{dn}{dt}=q-\alpha n^2-\eta Nn$$

where it is assumed that the number of positive and negative ions are equal, and they recombine with the recombination coefficient α while all attachment to more immobile particles of volume density, N, takes place with the attachment coefficient η. The accepted value for α is 1.6×10^{-6} cm.^{3} sec.$^{-1}$, and Chalmers [2] has an average value for η of 2×10^{-6} cm.^{3} sec.$^{-1}$ Some of the cases for the solutions of this equation in a thunderstorm have been discussed by Freier [5].

In a region of space where N is much less than n, the equilibrium value for n is $\sqrt{q/\alpha}$. At the 500-mb. level of pressure in the earth's atmosphere, cosmic rays produce ions at the rate of $q=15.4$ ion pairs cm.$^{-3}$ sec.$^{-1}$ The corresponding equilibrium value of $n_{\pm}$ is 3100 cm.$^{-3}$, and if the singly charged ions had a mobility of $\omega=1$ cm.^{2} sec.$^{-1}$ v.$^{-1}$, the conductivity would be $\lambda=2ne\omega=8.9\times10^{-4}$ sec.$^{-1}$ A charge placed in this medium would decay with a relaxation time of $\tau=1/4\pi\lambda=90$ sec. If the source of cosmic radiation were suddenly removed, the number of ions would decrease from the equilibrium value as

$$n=\sqrt{\frac{q}{\alpha}}\frac{1}{(1+\sqrt{q\alpha}t)}.$$

In a time $t=1/\sqrt{q\alpha}$, n would decrease to half the original equilibrium value, and for the above values of q and α this characteristic time would be 200 sec. For times much less than 200 sec.,

$$\lambda\simeq2\sqrt{\frac{q}{\alpha}}\,e\omega(1-\sqrt{q\alpha}t),$$

and a charge placed in the medium would decay with an observed relaxation time of 90 sec.

If by contrast, the original source strength were due to corona discharge in strong electric fields such that $q=10^4$ cm.$^{-3}$ sec.$^{-1}$, then the equilibrium values for n_1 would be 7.9×10^4 cm.$^{-3}$, the corresponding conductivity for small ions would be $\lambda=2.3\times10^{-2}$ sec.$^{-1}$, and the relaxation time for a charge in the medium would be 3.5 sec. If the corona discharge suddenly stopped due to an electrical discharge which reduced the field causing the corona, the number of ions would reduce to half the original value in a time of 8.0 sec. An observer studying the relaxation time of the electric field due to a charge placed in the medium would find a relaxation time of 3.5 sec., even though the production rate had gone to zero. Ohm's law in a most strict sense would not hold during the entire process, yet the short relaxation time would be indicative of a large conductivity.

In a thunderstorm the environment is altered by the presence of N larger particles per unit volume which can capture the small ions (Fletcher [4]). The electric field may or may not be involved in the capture of ions. If the electric field is involved we probably have a case of hyperelectrification described by Gunn [7] and if the field is not involved we can use the recombination coefficient, η, given above.

For the case for which the electric field plays no role it may be assumed for simplicity that $N>>n$. The equilibrium value for n is then $n_1=q/\eta N$. If $N=5\times10^3$ cm.$^{-3}$ and if q is determined by cosmic ray ionization, then the equilibrium value is $n_1=15.4/(2\times10^{-6})(5\times10^3)=1.5\times10^3$ cm.$^{-3}$ The corresponding electrical conductivity would be determined almost entirely by the small ions which have the equilibrium value n_1 and thus give a value for $\lambda=4.4\times10^{-4}$ sec.$^{-1}$ and a relaxation time of 180 sec. Wilson [12] and Gunn [6] believed that a situation somewhat similar to this exists in the thunderstorm cloud.

If corona discharge could increase q to 10^4 cm.$^{-3}$ sec.$^{-1}$ and N were also increased to 10^5 cm.$^{-3}$, then the equilibrium value for small ions would be $n_1=5\times10^4$ cm.$^{-3}$, and the values for λ and τ would be respectively 1.4×10^{-2} sec.$^{-1}$ and 5.7 sec. These values are close to those given by Latham and Mason [10] for conductivity of the air in a thunderstorm where there is corona from the drops.

If the corona discharge were suddenly stopped for the case where $N>>n$, the value of the number of small ions would decrease at the rate

$$n=\frac{q}{\eta N}\exp(-\eta Nt).$$

The small ion density, n, would decrease by a factor of e in $1/\eta N=5$ sec. This time is now less than the relaxation time calculated above, so that the observed relaxation rate for a charge in the medium would deviate seriously from a simple exponential form.

The other case to be considered for thunderstorms is that where the hyperelectrification described by Gunn [7] is used to capture ions. If the production of ions is by corona discharge from the drops, the conductivity will most likely be unipolar, and consequently the drop may become charged by the corona produced ions. When a spherical drop of radius a and charge Q' is placed in an electric field, it will have a current

$$I'=3E_0a^2\pi ne\omega(1-Q'/Q_m)^2$$

flowing to it. Q_m is the maximum charge ($Q_m=3a^2E_0$) that the drop can have and still have field lines terminate on it. If there are N drops per cm.^{3} the number of ions captured per unit time in a unit volume is $3E_0a^2\pi n\omega(1-Q'/Q_m)^2N$.

The electric field at the opposite side of the drop where field lines again originate is $E'=3E_0(1+Q'/Q_m)$, and it is here that corona would take place. N drops per cm.^{3} would give a source strength of ions, q, such that at equilibrium

$$q=3E_0a^2\pi n\omega(1-Q'/Q_m)^2N.$$

The number of droplets, N, per unit volume is limited by the water content, W, of the cloud. If ρ is the mass density of water, the equilibrium condition may then be written as

$$\frac{q}{n}=\frac{9}{4}\frac{E_0}{a}\frac{\omega}{\rho}(1-Q'/Q_m)^2W.$$

For a field of 1 stat. v./cm., a water content of 5 gm. m.$^{-3}$, drops of 10^{-3}-cm. radius, and an ion mobility of 300 cm.^{2} sec.$^{-1}$ (stat. v.)$^{-1}$ this becomes

$$\frac{q}{n}=3.4(1-Q'/Q_m)^2 \text{ sec.}^{-1}$$

The charge on the drops can change the relation between the production rate and the equilibrium number of ions, and for a given charge on the drops n must be large if q is large.

In the case of hyperelectrification the charged droplets can also be moved by the electric field to give a conduction current. The electric force on the droplet would be E_0Q', and this force can be in equilibrium with the viscous force $6\pi a\eta' v$ if drops are sufficiently small so that gravitational forces may be neglected. The current density due to motion of the drops in the field is then

$$NQ'v=\frac{NQ'^2E_0}{6\pi a\eta'}$$

where v is the velocity of the drop and η' is the viscosity of the air. N is limited by the liquid water content and Q' may be expressed as a fraction, f, of the maximum charge. When these substitutions are made the conduction current density due to the motion of the drops becomes

$$NQ'v=\frac{9}{8\pi^2}f^2\frac{E_0^3W}{\rho\eta'}.$$

The viscosity of air is 1.8×10^{-4} dyne sec. cm.$^{-2}$ so that with a field of one stat. v.cm.$^{-1}$ the current density is $j=3f^2 \times10^{-3}$e.s.u. cm.$^{-2}$ sec.$^{-1}$ which is quite large for f near unity. For larger fields this current would be considerably greater. If this current dissipates other cloud charges a relaxation time of $\tau=E_0/4\pi NQ'v$ would be observed and for the above choice of values this would yield $\tau=(26.3/f^2)$ sec.

There seems to be evidence that hyperelectrification can lead to corona and short relaxation times but nothing very quantitative can come from the present knowledge of the processes involved. This laboratory is making studies of the processes described above, but we find the measurements difficult to make. With corona discharge in a field of condensed water droplets the space charge gives such a large divergence to the electric field that it is difficult to know just where in the region the above conditions are satisfied.

This large divergence of the electric field would also be present in a thunderstorm so that selected conditions could prevail only in a relatively small region of space. However, the process described is one in which the deposition of charge could progress through the medium in the direction of the field and thus deposit charge in large regions of the thunderstorm volume.

If there were no corona in the thunderstorm cloud the above equations would have $Q'=0$ and $f=0$ so that the droplets would be efficient sinks for all ions in the presence of an electric field. In this case the conductivity of the air would be very small and relaxation times would be very long. This author thinks that this possibility is incompatible with observations of relaxation times made in thunderstorm processes.

After a lightning discharge it seems that the electric field may be in general sufficiently weak so that hyperelectrification need not be considered in the early stages of field regeneration. The early stages of field regeneration and observed relaxation times for field recovery should then reflect processes of ordinary ion attachment.

3. CONDUCTION BY EDDY DIFFUSION

Turbulence of the air may also play some role in the decay of charge and the associated fields. In the cases of conduction as discussed above, the detailed mechanism is one of forced molecular diffusion which leads to a relation between the molecular diffusion constant and the ion mobility, namely, $\omega=eD/kT$, where ω is the ion mobility, D is the molecular diffusion constant, k is Boltzmann's constant, and T is the absolute temperature. Kennard [9] shows how the forced diffusion can always lead to an equivalent density gradient. The settling of a fine precipitate such as found in Perrin's experiment as described by Block [1] is a case where the forced diffusion due to gravity is in equilibrium with the diffusion due to density gradients. The diffusion of oxygen and nitrogen in the earth's atmosphere destroys gravitational stratification.

The statistical arguments for random walk in molecular diffusion processes are given by Joos [8] and lead to the result

$$\frac{\partial n}{\partial t}=\frac{X^2}{2T'}\cdot\frac{\partial^2 n}{\partial x^2}=D\frac{\partial^2 n}{\partial x^2} \tag{1}$$

where X^2 is the mean square displacement for a particle in the time T' when subject to a concentration gradient $\partial n/\partial x$. It appears that similar statistical arguments may be made for eddy diffusion, in which case the mean square displacements are 10^5 times greater than for molecular diffusion (Sutton [11]).

If the electrical force on a particle is $\mathbf{F}=\mathbf{E}e$, where $\mathbf{E}$ is the electric field, the particle will be accelerated until it encounters an interaction where it gives up energy. Repeated accelerations and interactions lead to a drift velocity $\mathbf{V}$, which is proportional to $\mathbf{F}$ or $\mathbf{V}=K\mathbf{F}=Ke\mathbf{E}$, where K is a constant. This drift velocity will produce a flux of particles equal to $n\mathbf{V}=nK\mathbf{F}$. If this electrical forced diffusion were in equilibrium with diffusion produced by concentration gradients the fluxes would be equal and opposite so that the first integral of equation (1) at equilibrium would be

$$-D\frac{\partial n}{\partial x}=nKF=nKeE. \tag{2}$$

The quantity Ke would be the mobility, ω, of the ion, and D is the coefficient of diffusion. In the case of molecular diffusion the ions would have a Boltzman distribution in the electrostatic potential, V, given by $n=n_0 \exp(-Ve/kT)$, and in this case

$$\frac{1}{n}\frac{\partial n}{\partial x}=-\frac{e}{kT}\frac{\partial V}{\partial x}=\frac{eE}{kT},$$

where the electric field is given by $E=-\partial V/\partial x$. The dynamic mobility of this ion in molecular diffusion is then $\omega=Ke=eD/Tk$.

In the case of eddy diffusion, the same arguments could be used to derive equations (1) and (2) with the coefficient of eddy diffusion, D', replacing the coefficient of molecular diffusion, D.

This author has not been able to find a statistical relation for $(1/n)\partial n/\partial x$ in the case of eddies and turbulent motion. If this value exists and if it is sufficiently large, an ion could have a large mobility due to processes of eddy diffusion.

The experiments of Colgate [3] imply a large electrical conductivity in the cloudy region of a diffusion cloud chamber where there could be turbulent motion due to bubbling of carbon dioxide escaping from water. One would conclude from the experiment that eddy diffusion gave a large electrical conductivity.

The author has done experiments in which charges were placed on well-insulated stator blades of an electric field mill. The charge could be conducted from these plates through the surrounding air. When the field mill is shielded from the surroundings, it will then measure its own field, and the decay rate can be measured.

The experiment was done by running the mill continuously in some cases and by running it intermittently in other cases. Very little difference was noted in the decay rate or relaxation time even though in the case of continuous running there would be considerable turbulence around the blades throughout the decay period. In all cases the relaxation time corresponded to that given by molecular conductivity, and it was concluded that eddy diffusion plays a very minimum role in charge transport.

4. CONCLUSIONS

Even though electric fields in thunderstorms may produce corona discharges intermittently to give a non-linear relationship between the current density and the electric field, the relaxation times for the electric field give a true indication of whether or not there is a large value of the electrical conductivity. The relaxation times for the changes in the electric field may be altered by the characteristic times for the ions to recombine or attach to more immobile particles. As long as the electric field is sufficiently large to maintain a corona discharge within the cloud, the equilibrium value of small ions can be maintained at a sufficiently high level to give a high electrical conductivity, and this high electrical conductivity can persist for times which are as long or, in most cases, longer than the relaxation time of the medium for field changes. A short relaxation time for the electric field changes necessarily implies a large value for the electrical conductivity of the medium.

It is not clear whether eddy diffusion can contribute to the electrical conductivity.

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